Report on

Flow through a Convergent-Divergent Nozzle

Case Study Project

Under the guidance of
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1. Introduction

Consider a subsonic flow through a convergent nozzle. The quasi-one-dimensional equations (section 2.1) confirm that the flow speeds up along the nozzle, reaches sonic speed and then slows down. Therefore, for a gas to expand isentropically from subsonic to supersonic speeds, it must flow through a convergent-divergent duct. The minimum area, called throat, that divides the convergent and divergent sections of the duct has sonic flow. Rocket engines are primarily convergent-divergent nozzles which expand the exhaust gases to high-velocity, supersonic speeds. A convergent-divergent nozzle is sometimes called a de Laval (or Laval) nozzle. Depending upon the geometry and exit conditions, the flow through the convergent-divergent nozzles have different characteristics.

2. Governing Equations

The Navier-Stokes equations for an inviscid compressible flow in an arbitrary domain is

$$\frac{\partial(\rho\vec{u})}{\partial t} + \nabla.[\vec{u}(\rho\vec{u})] + \nabla p = 0$$

where all symbols have their usual meaning. The Navier-Stokes equation is supplemented with the conservation of mass

$$\frac{\partial\rho}{\partial t} + \nabla.(\rho\vec{u}) = 0$$

Conservation of total energy for an inviscid compressible flow gives

$$\frac{\partial(\rho E)}{\partial t} + \nabla.[\vec{u}(\rho E)] + \nabla.(p\vec{u}) = 0$$

where the total energy density $E = e + |\vec{u}|/2$ with e the specific internal energy.

The 3 equations are supplemented with an equation of state which is the isentropic relation

$$\frac{dp}{d\rho} = \left(\frac{\partial p}{\partial\rho}\right)_s = a^2$$

where a is the speed of sound.

2.1. Quasi-One-Dimensional Flow

The flow through a variable-area duct is three-dimensional in reality. But with the quasi-one-dimensional assumption, the flow through the area-variable duct varies only as a function of x, i.e., $u = u(x)$, $p = p(x)$, etc. This assumption that flow properties are uniform across any given cross section represent values that are some kind of mean of the actual flow properties distributed over the cross section clearly shows that the quasi-one-dimensional flow is an approximation to the actual physics of the flow.

Consider an incremental volume as shown in fig. 1.

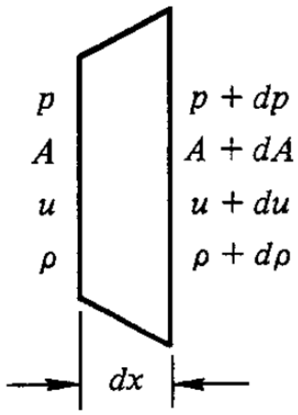

Figure 1. Incremental Volume

Considering this infinitesimal control volume for conservation of mass, momentum and energy equation, and a few algebraic simplifications we get

$$d(\rho u A) = 0, \tag{1}$$

$$dp = -\rho u \, du, \tag{2}$$

$$dh + u \, du = 0 \tag{3}$$

where h is the specific enthalpy.

Equations (1) and (2) along with the isentropic relation gives

$$\frac{dA}{A} = (M^2 - 1)\frac{du}{u} \tag{4}$$

Equation (4) is called the area-velocity relation. It can be inferred from Equation (4) that for a gas to expand isentropically from subsonic to supersonic speeds, it must flow through a convergent-divergent duct.

Depending on the exit conditions, the flow through a convergent-divergent nozzle can be categorised in 3 broad cases. The results from each case are discussed without proof. Derivation is beyond the scope of this case study.

2.1.1. Isentropic Expansion from Subsonic to Supersonic Speeds

When the inlet pressure is specified, but no details about the exit conditions are known except that $p_{exit} < p_{in} = p_o$. The subsonic flow at the inlet expands isentropically, in the convergent section, to sonic speed at the throat where it expands, in the divergent section, further to supersonic speed at the exit. The flow properties are governed by the isentropic relation.

The variation of Mach number and pressure along the nozzle is shown in fig. 2. At the throat, the flow is sonic. Hence, denoting conditions at sonic speed by an asterisk (*).

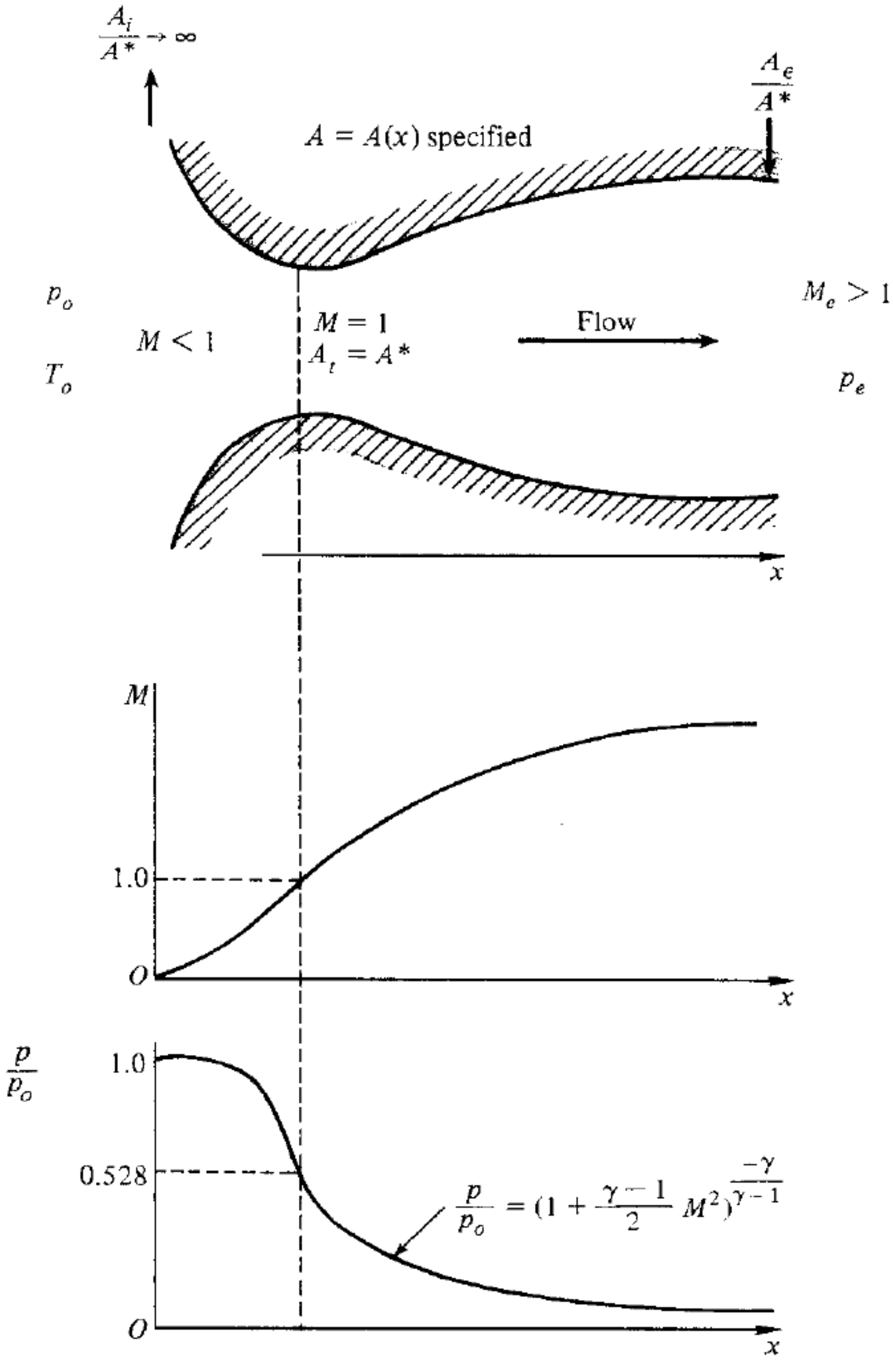

Figure 2. Isentropic supersonic nozzle flow [1].

2.1.2. Isentropic Subsonic Flow

When the inlet and exit pressure is specified and the pressure ratio is weak, i.e., $p_{exit}/p_{in} < 1$ and also close to 1. The subsonic flow from inlet speeds up as it reaches the throat. At the throat the flow is still subsonic. The flow then slows down along the divergent section until exit. The flow remains subsonic throughout the nozzle.

The variation of Mach number and pressure along the nozzle is shown in fig. 3. At the throat, the flow is still subsonic. Therefore, there are no asterisk properties. The local Mach number reaches the maximum at the throat, where the cross-sectional area is the minimum, denoted by A_t.

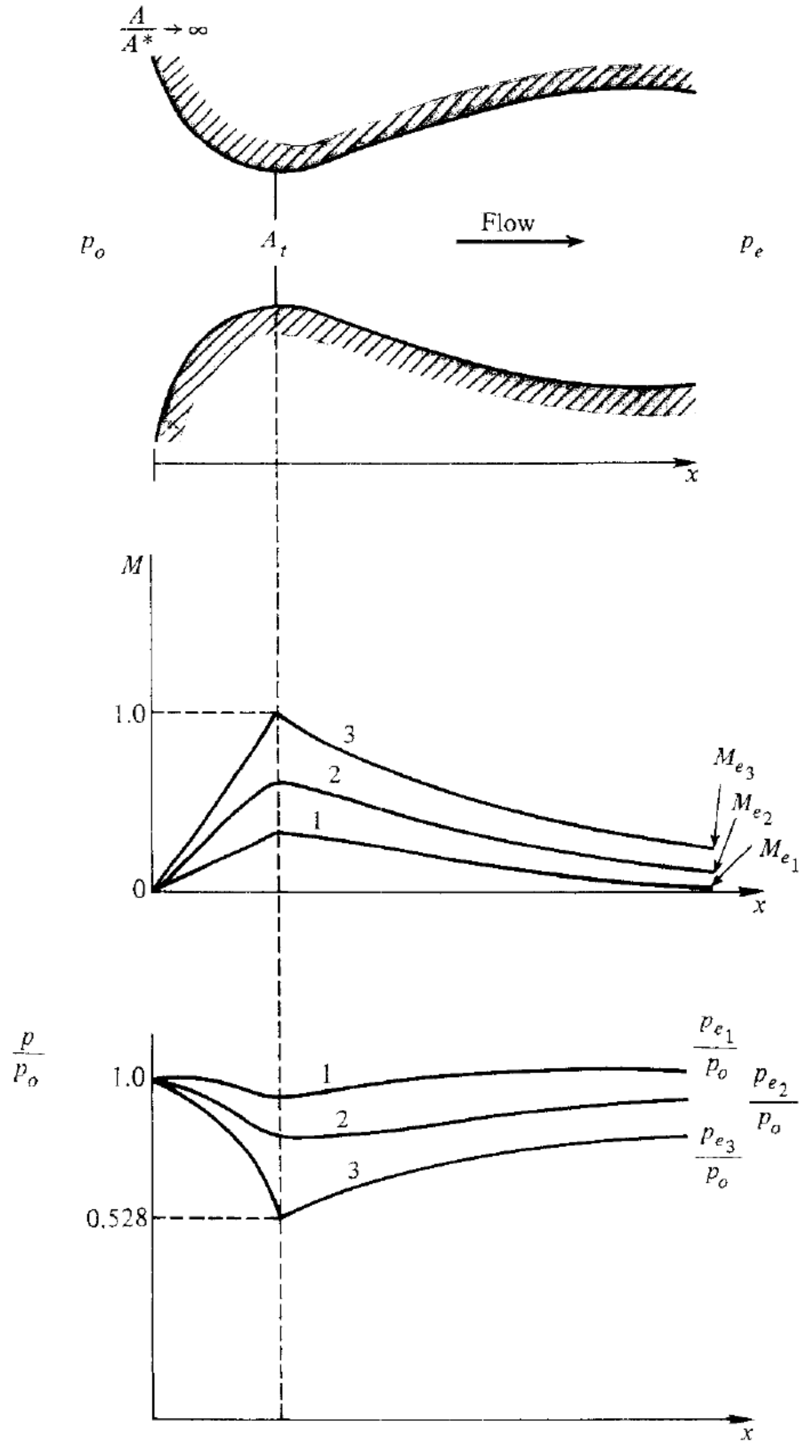

Figure 3. Subsonic flow in a convergent-divergent nozzle [1].

2.1.3. Supersonic Flow with a Normal Shock

When the inlet and exit pressure is specified and the pressure ratio is strong, i.e., $p_{exit}/p_{in} < 1$ and also not close to 1. The subsonic flow at the inlet expands isentropically, in the convergent section, to sonic speed at the throat. A normal shock wave exists inside the divergent duct. This situation is shown in fig. 4. Isentropic relations are not valid in this section.

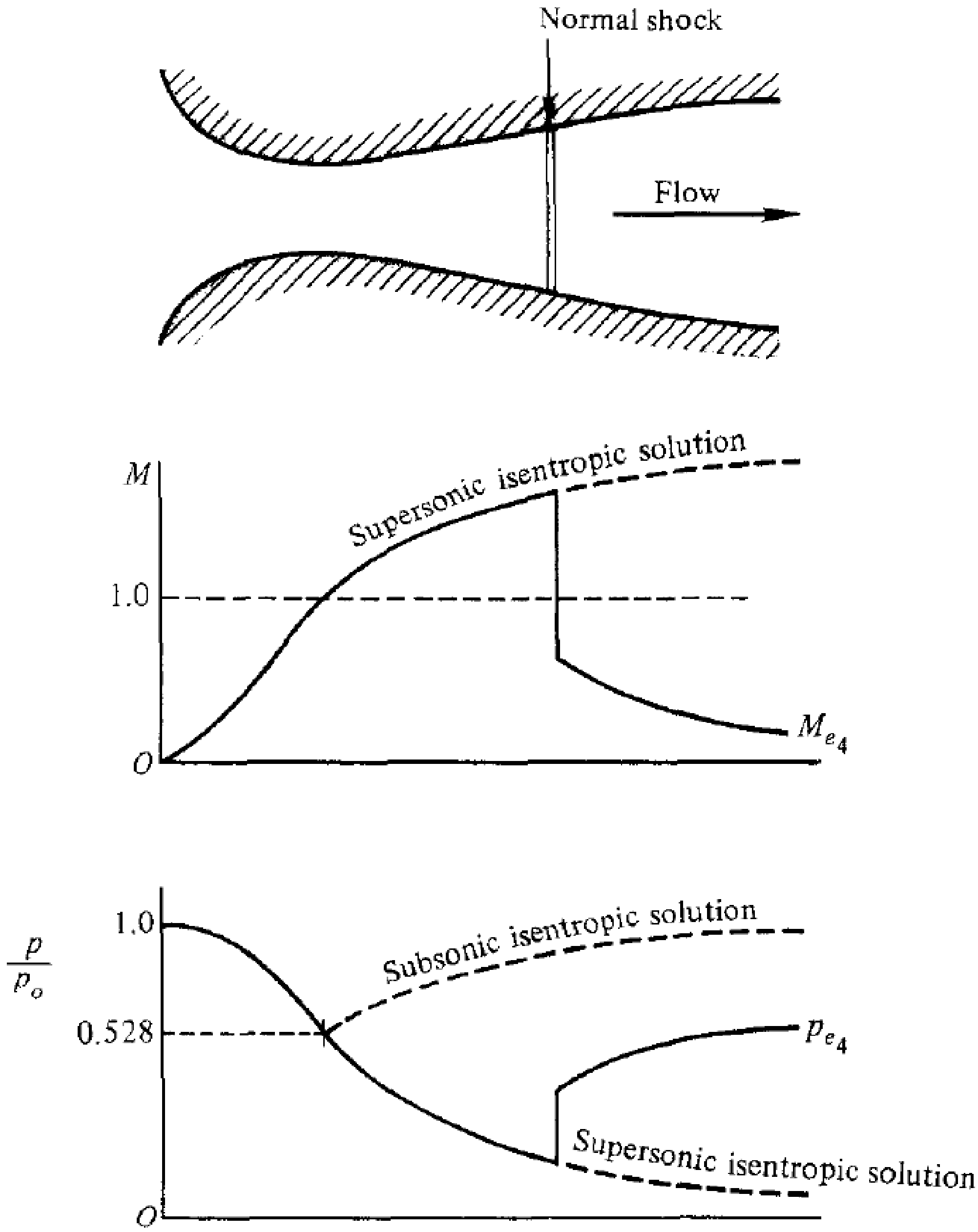

Figure 4. Flow with a shock wave inside a convergent-divergent nozzle [1].

The location of the normal shock wave in the duct is determined by the requirement that the increase of static pressure across the wave plus that in the divergent portion of the subsonic flow behind the shock be just right to achieve p_{e4} (as in fig. 4), at the exit. As the exit pressure is reduced further, the normal shock wave will move downstream, closer to the nozzle exit.

3. Implementation in OpenFOAM

3.1. Problem Statement

The problem considers flow of air through a convergent-divergent nozzle. The inlet pressure (p_{in}) and temperature is 10000 Pa and 298 K respectively.

The simulation is run for 3 different exit conditions

1. $p_{exit} = 1600$ Pa. Isentropic Expansion from Subsonic to Supersonic Speeds.
2. $p_{exit} = 8900$ Pa. Isentropic Subsonic Flow.
3. $p_{exit} = 7500$ Pa. Supersonic Flow with a Normal Shock.

3.2. Geometry & Meshing

A convergent divergent nozzle of geometry as shown in fig. 5 is considered. The nozzle cross-section varies as

$$A(x) = \begin{cases} 1.75 - 0.75\cos(0.2x - 1)\pi, & 0 < x \leq 5 \\ 1.25 - 0.25\cos(0.2x - 1)\pi, & 5 < x \leq 10 \end{cases}$$

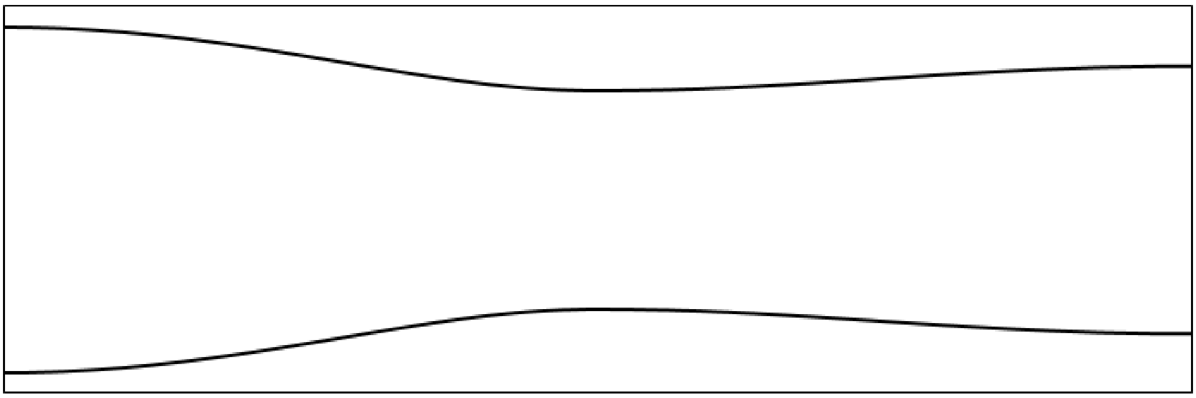

Figure 5. The configuration of flow through a convergent-divergent nozzle.

The geometry is an axi-symmetric nozzle, therefore a wedge is considered for the analysis. The geometry was created using blockMesh utility. The meshing is simpleGrading.

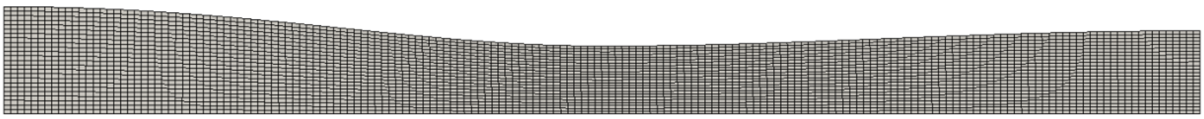

Figure 6. Meshing of the wedge.

3.3. Initial & Boundary Conditions

Case 1. Isentropic Expansion from Subsonic to Supersonic Speeds		
Pressure	Inlet	10000 Pa
	Nozzle	Zero Gradient
	Outlet	1600 Pa
Velocity	Inlet	Zero Gradient
	Nozzle	Slip
	Outlet	Zero Gradient
Temperature	Inlet	298 K
	Nozzle	Zero Gradient
	Outlet	Zero Gradient

Case 2. Isentropic Subsonic Flow		
Pressure	Inlet	10000 Pa
	Nozzle	Zero Gradient
	Outlet	8900 Pa
Velocity	Inlet	Zero Gradient
	Nozzle	Slip
	Outlet	Zero Gradient
Temperature	Inlet	298 K
	Nozzle	Zero Gradient
	Outlet	Zero Gradient

Case 3. Supersonic Flow with a Normal Shock		
Pressure	Inlet	10000 Pa
	Nozzle	Zero Gradient
	Outlet	7500 Pa
Velocity	Inlet	Zero Gradient
	Nozzle	Slip
	Outlet	Zero Gradient
Temperature	Inlet	298 K
	Nozzle	Zero Gradient
	Outlet	Zero Gradient

Since the flow is axi-symmetric, both the wedge faces are assigned ‘wedge’ boundary condition.

3.4. Solver

The flow through convergent-divergent nozzle governing equations, as described in section 2, are solved using rhoCentralFoam [2]. The thermophysical properties of air, assuming perfect gas, is used. The simulation type is laminar.

4. Results

The simulations are run on OpenFOAM 5.0 and the post processing is done using ParaView.

4.1. Case 1: Isentropic Expansion from Subsonic to Supersonic Speeds

The pressure field at steady-state in the xy-plane is shown in fig. 7.

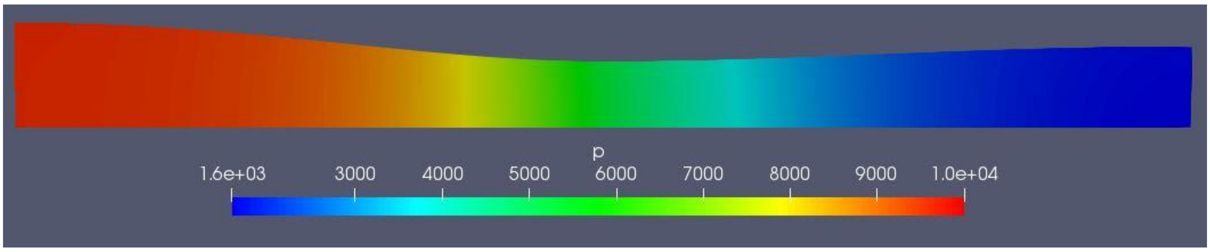

Figure 7. Steady-state pressure field.

The Mach number and pressure variation along the nozzle is shown in fig. 8 and fig. 9 respectively.

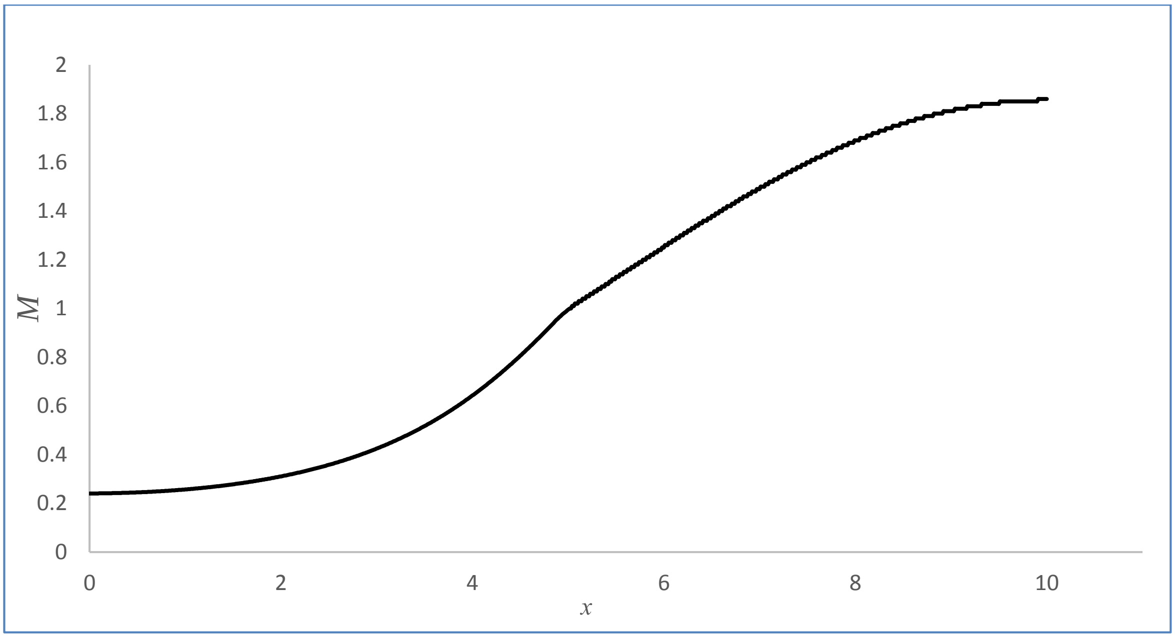

Figure 8. Variation of Mach number along the nozzle.

The pressure and Mach number profiles match with the theoretical observation discussed in section 2.1.1.

The flow, as indicated in fig. 8, reaches sonic speed at the throat, from where it expands to supersonic speeds at the exit.

The pressure variation, as indicated in fig. 9, also follows the isentropic relation throughout the nozzle.

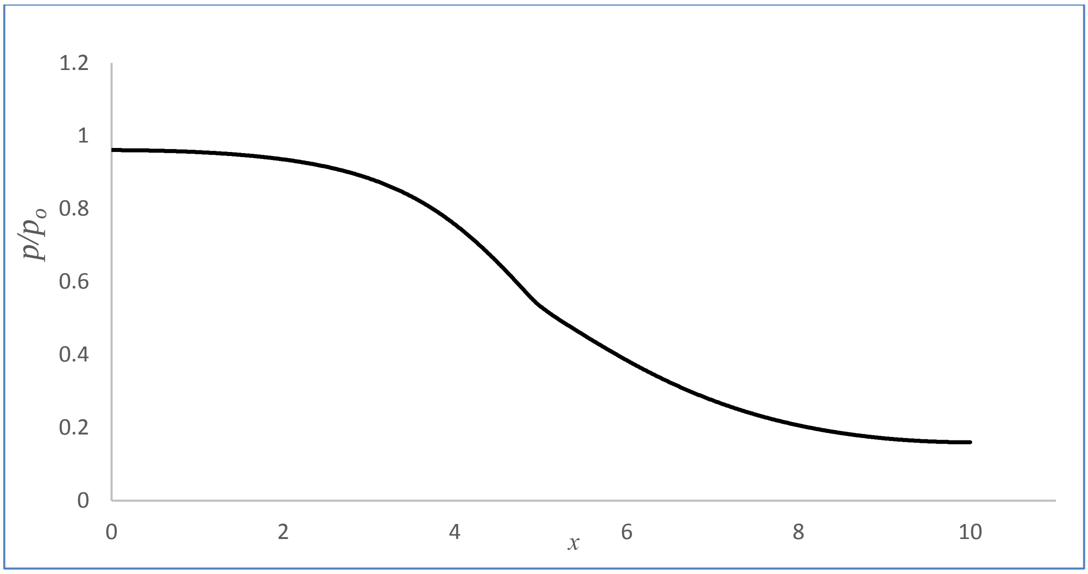

Figure 9. Variation of pressure along the nozzle.

4.2. Case 2: Isentropic Subsonic Flow

The pressure field at steady-state in the xy-plane is shown in fig. 10.

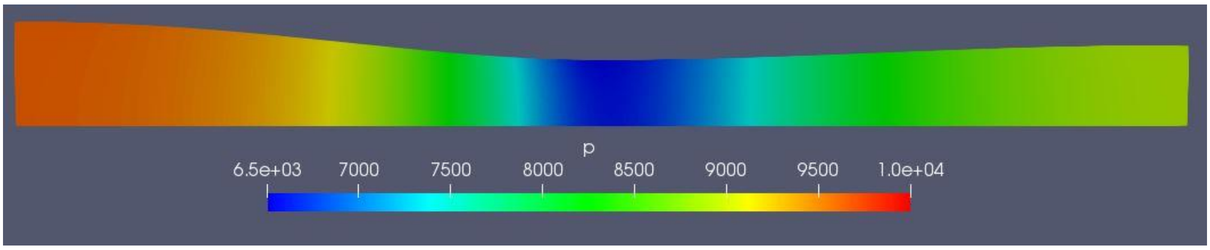

Figure 10. Steady-state pressure field.

The Mach number and pressure variation along the nozzle is shown in fig. 11 and fig. 12 respectively.

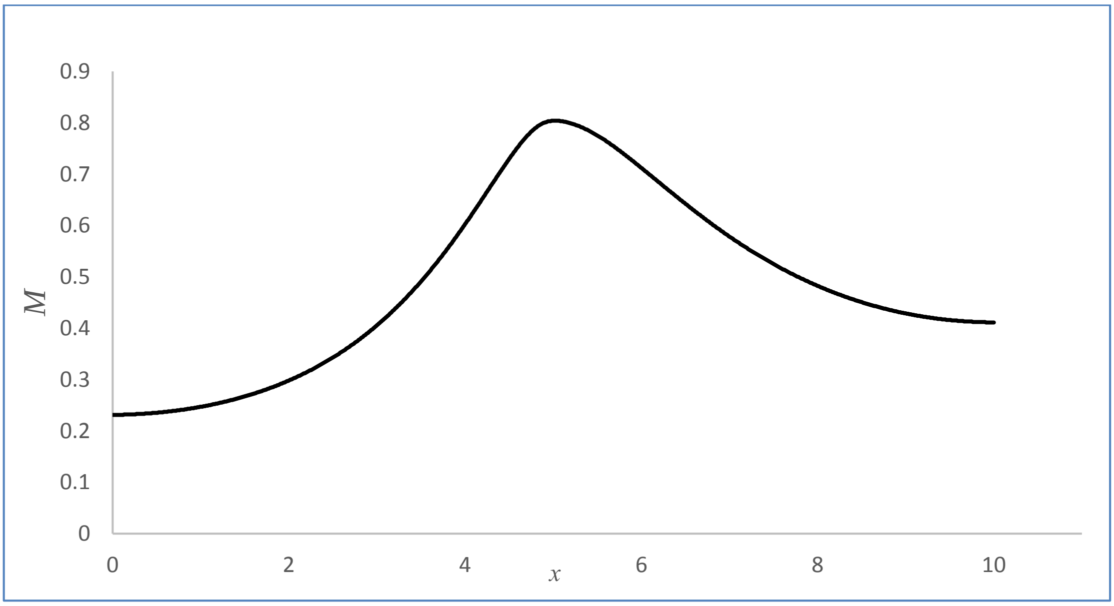

Figure 11. Variation of Mach number along the nozzle.

The pressure and Mach number profiles match with the theoretical observation discussed in section 2.1.2.

The flow, as indicated in fig. 11, remains subsonic throughout the nozzle.

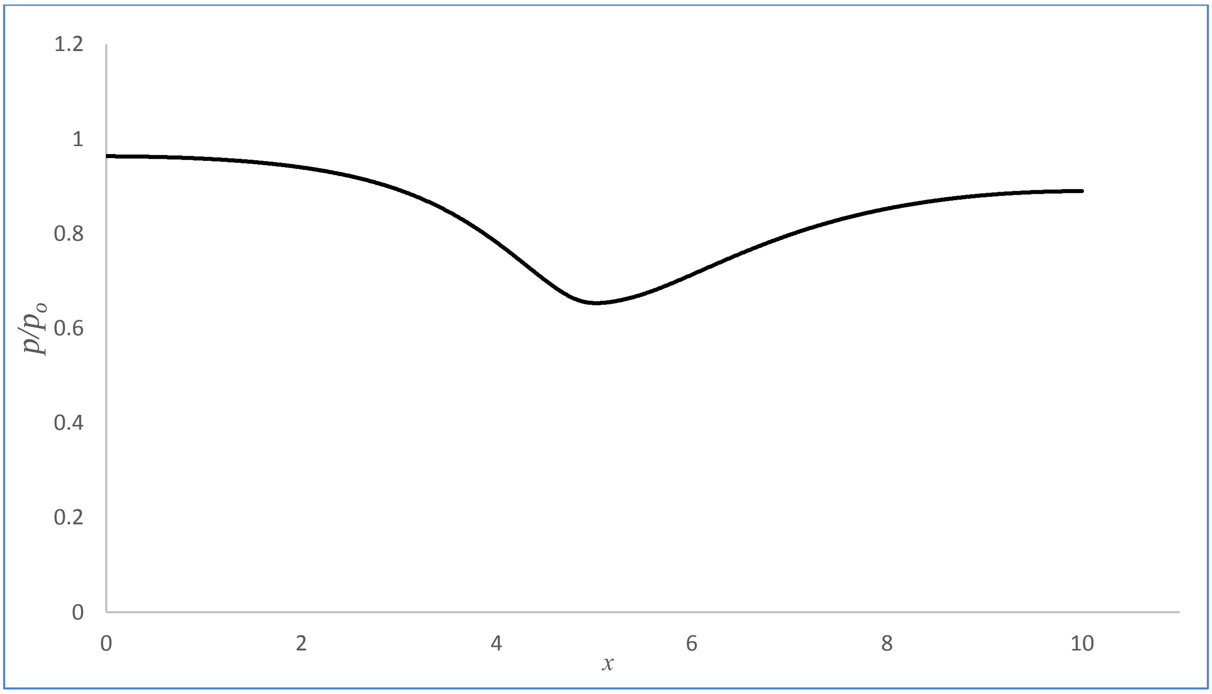

Figure 12. Variation of pressure along the nozzle.

4.3. Case 3: Supersonic Flow with a Normal Shock

The pressure field at steady-state in the xy-plane is shown in fig. 13.

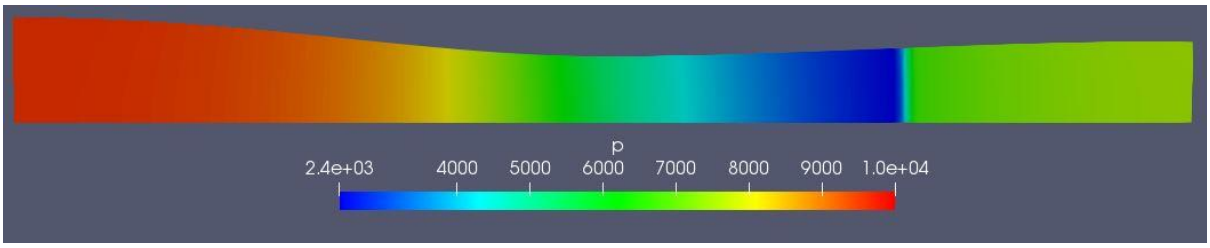

Figure 13. Steady-state pressure field.

The Mach number and pressure variation along the nozzle is shown in fig. 14 and fig. 15 respectively.

The pressure and Mach number profiles match with the theoretical observation discussed in section 2.1.3.

The flow, as indicated in fig. 14, reaches sonic speed at the throat, from where it expands to supersonic speed till the location of normal shock. After the shock the flow slows down and remains subsonic for the latter section of the nozzle.

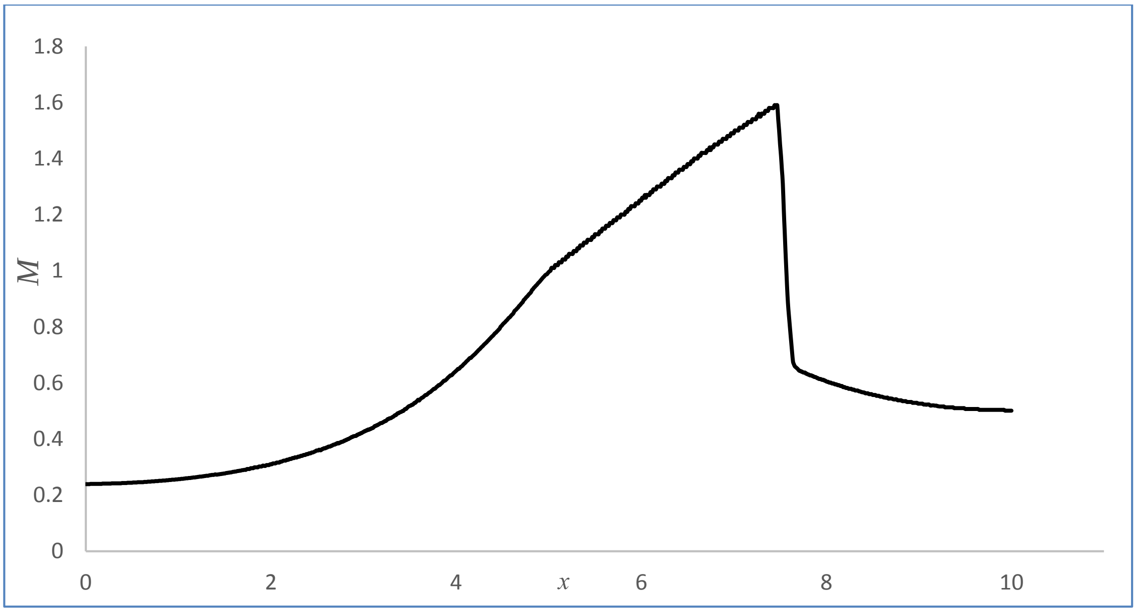

Figure 14. Variation of Mach number along the nozzle.

The shock is located at $x = 7.5$. This result agrees well with the analytical solution ($x = 7.5623$).

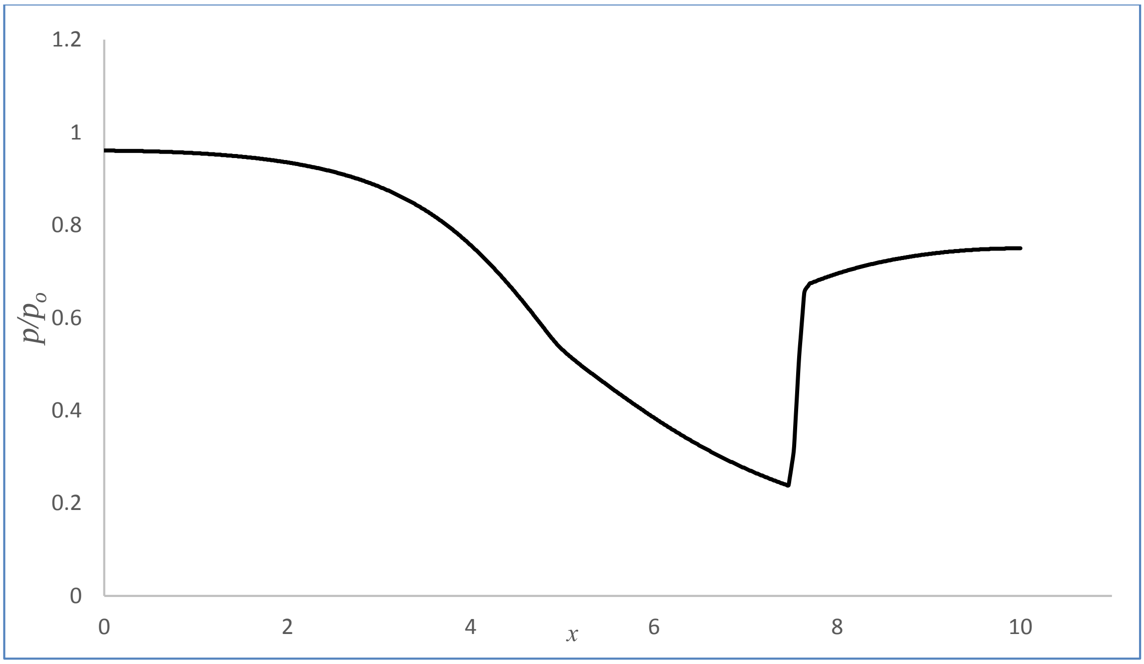

Figure 15. Variation of pressure along the nozzle.

5. Conclusion

The flow of air through a convergent-divergent nozzle using OpenFOAM solver rhoCentralFoam. The simulation produced expected result. The results, especially in the supersonic flow with normal shock, could have been captured better with a more refined mesh. Adaptive meshing could also be used to capture the shock better. The problem used for simulation is taken from NASA CFD benchmark case [3]. The simulated results match well with analytical solution.

References

1. Anderson, J.D., Modern Compressible Flow, McGraw Hill Inc., New York, 1984.

2. Greenshields, C. J., Weller, H. G., Gasparini, L. and Reese, J. M. (2010), Implementation of semi-discrete, non-staggered central schemes in a collocated, polyhedral, finite volume framework, for high-speed viscous flows. Int. J. Numer. Meth. Fluids, 63: 1-21. doi:10.1002/fld.2069

3. NPARC Alliance Verification and Validation Archive. Steady, Inviscid Flow in a Converging-Diverging Verification (CDV) Nozzle.