Validation of Stokes Law

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Abstract:

The report aims to validate the Stokes' Law for frictional or drag force on spherical objects for laminar flow using the software ANSYS and OpenFOAM. This is aimed to be achieved by comparing the numerical results obtained for Stokes' Law and the analytical results obtained from OpenFOAM. Multiple samples will be created and will be compared against their respective results. Though Stokes' Law is simple and only applicable for laminar flow, it has many important applications such as the study of the motion of microorganisms, water droplet falling from the cloud, etc. This makes it necessary to analyse it to find the good working range of this law.

Keywords- Stokes law, simpleFoam, OpenFoam, Paraview.

1. Introduction

This report is based on the analysis on a sphere moving with its terminal velocity in a given fluid. The numerical equation to find the frictional force – also called drag force– exerted on spherical objects with very small Reynolds Number in a viscous fluid was given by **Sir George Stokes** in 1851 [1]. This equation is when the terminal velocity is achieved by the body. This equation is based on a few assumptions and has a very low domain of application, but it makes calculations simple and saves the cost and time required for complex calculations.

I] Geometry

The geometry was created using software ANSYS 16.0. The diameter of the sphere is 2mm and is placed in the domain of 60mm x 30mm x 30mm (length x width x height).

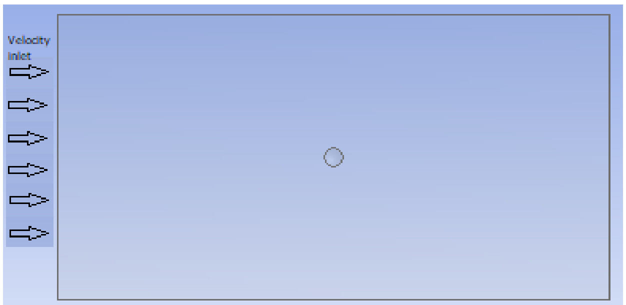

Fig a: Spherical body in fluid domain considered for computation

II] Mesh

The meshing for this simulation was done using the software ANSYS 16.0. Mesh quality and aspect ratio was optimized using ANSYS 16.0.

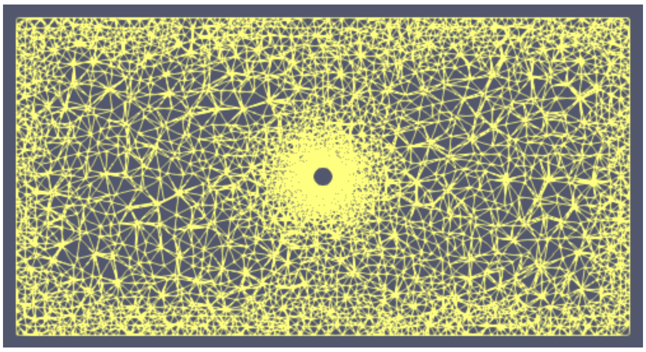

Fig b: Mesh on slicing plane XY.

Note: The centre of gravity of spherical body coincides with the origin (0 0 0).

2. Analysis

The CFD analysis of the flow of inviscid fluid (Glycerine) over the spherical body was done using the software OpenFOAM-v1812.

I] Boundary Conditions

To verify Stokes Law, different values of Reynolds number ranging from 0 to 1 were taken. For each different value, the different simulation was performed. Glycerine was considered for this simulation. Other fluids can also be used but the values of constants and velocities will change accordingly.

The condition for inlet velocity of Glycerine is different for all the cases and can be calculated with the formula

$$U = \frac{Re\ .\ \nu}{L} \qquad (1)$$

Where,

U - Inlet velocity/ free stream velocity
Re - Reynolds number
ν - Kinematic viscosity
L - Characteristic linear dimension

The values of Reynolds number are taken from 0 to 1 because Stokes law is used for Re<1.

The free stream velocity at the inlet was set to be normal to inlet surface. The pressure was set to be zero gradients at the fluid interface. The spherical surface was defined as a wall with no slip. The fluid interface was modelled as a slip. The outlet was kept fixed as zero gradients.

II] Constants

To carry out this analysis, the properties of Glycerine were used.

At 20 ºC,

The kinematic viscosity of Glycerine = 648 centistokes
= 648 x 10^{-6} m^2/s

Density of Glycerine = 1260 kg/m^3

III] Model

Laminar model of OpenFOAM is used for this simulation. Turbulence was turned off.

IV] simpleFoam

Though simpleFoam is steady state turbulent solver for incompressible flows, it can be used for laminar. When RASModel is set to be laminar, it doesn't read wall functions.

The solver follows a segregated solution strategy. This means that the equations for each variable characterizing the system (the velocity U, the pressure P and wall functions if turbulence is on) is solved sequentially and the solution of the preceding equations is inserted in the subsequent equation. The non-linearity appearing in the momentum equation (the face flux Φ which is a function of the velocity) is resolved by computing it from the velocity and pressure values of the preceding iteration. The dependence from the pressure is introduced to avoid a decoupling between the momentum and pressure equations. The procedure is repeated until convergence [2].

The equations used by simpleFoam to solve laminar incompressible flow are

Momentum equation

$$\frac{\partial (u_{rj} u_i)}{\partial x_j} + \epsilon_{ijk}\omega_i u_j = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \frac{1}{\rho}\frac{\partial}{\partial x_j}\left(\tau_{ij} + \tau_{t_{ij}}\right) \tag{2}$$

Pressure equation

$$\nabla \cdot \left(\frac{1}{a_p}\nabla p\right) = \nabla \cdot \left(\frac{H(\vec{U})}{a_p}\right) = \sum_f \vec{S}\left(\frac{H(\vec{U})}{a_p}\right)_f \tag{3}$$

Though there is very less chance of occurrence of vorticity and circulation in laminar flows, simpleFoam will still solve considering variables vorticity, circulation, etc.

The SIMPLE algorithm of simpleFoam is written below [3].

1) Start solving from boundary conditions.
2) Solve the momentum equation to find intermediate velocities.
3) Calculate the mass flux at each cell.
4) Solve the pressure equation.
5) Correct the mass fluxes.
6) Correct the velocities from the new values of pressure.
7) Update all the initial conditions.
8) Repeat till convergence.

V] Post Processing

The following commands was used to calculate the viscous drag. (**wallShearStress** calculates the viscous drag)

```
functions
{

  #includeFunc wallShearStress

}
```

3. Numerical Calculations

To calculate the drag force on the sphere, Stokes Law can be used. The law is stated as follows.

$$F_d = 6\pi\mu R V_T \tag{4}$$

where,

$\mathbf{F_d}$	– the Drag force
μ	– Viscosity of Glycerine
R	– Radius of the spherical body
$\mathbf{V_T}$	– Terminal velocity of the spherical body

The terminal velocity, in this case, is defined as the constant velocity of the spherical body inside the given fluid when two equal and opposite force act on it. As far as Stokes law is concerned, these forces are a gravitational force and buoyant force + drag force i.e.

gravitational force = buoyant force + drag force

In this simulation, the terminal velocity will be the same as free stream velocity.

4. Extraction of Data

The data was extracted using Paraview. The surface normals were generated by filters available in Paraview. After the generation of the surface normal, the following functions were calculated using a calculator feature available.

Function for pressure drag

p*Normals_X*1260 (5)

Function for viscous drag

wallShearStress_X * 1260 * sqrt((Normals_Y^2) + (Normals_Z^2)) (6)

where,

1260 is the density of Glycerine. It is needed to multiply by density because OpenFOAM calculated in the form of P/ρ that is, the dimensions of pressure here are L^2T^{-2}.

Integrate the results obtained from the above processes to get the total pressure and total viscous drag on the sphere.

Add both of the integrated results to get the total drag.

5. Results

Case no.	Re	U m/s	Total viscous drag (N)	Total pressure drag (N)	Total Drag (N)	Drag calculated by Stokes law (N)
1	0.1	0.0324	0.0003559	0.0001809	0.0005368	0.00049839
2	0.2	0.0648	0.0007132	0.0003627	0.0010759	0.00099678
3	0.3	0.0972	0.0010739	0.0005461	0.0016201	0.00149518
4	0.4	0.1296	0.0014387	0.0007318	0.0021705	0.00199357
5	0.5	0.1620	0.0018083	0.0009202	0.0027285	0.00249196
6	0.6	0.1944	0.0021835	0.0011117	0.0032952	0.00299035
7	0.7	0.2268	0.0025644	0.0013063	0.0038707	0.00348875
8	0.8	0.2592	0.0029512	0.0015043	0.0045556	0.00398714
9	0.9	0.2916	0.0033440	0.0017055	0.0050495	0.00448554
10	1	0.324	0.0037428	0.0019102	0.0056530	0.00498393

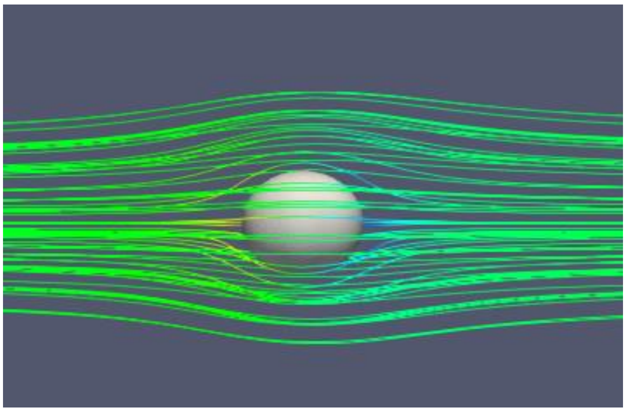

Fig c: Streamlines for case no. 10

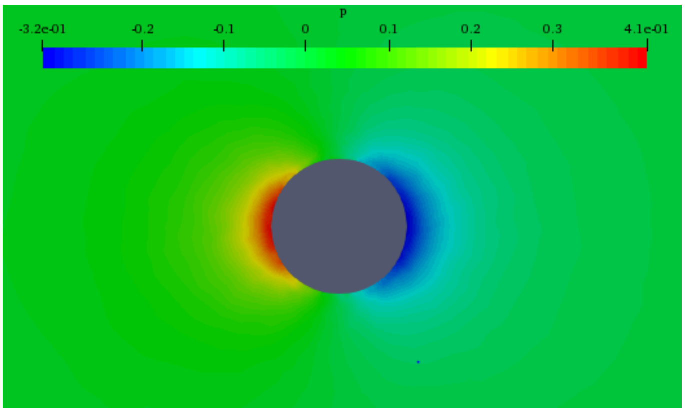

Fig d: Pressure variation for case no. 10

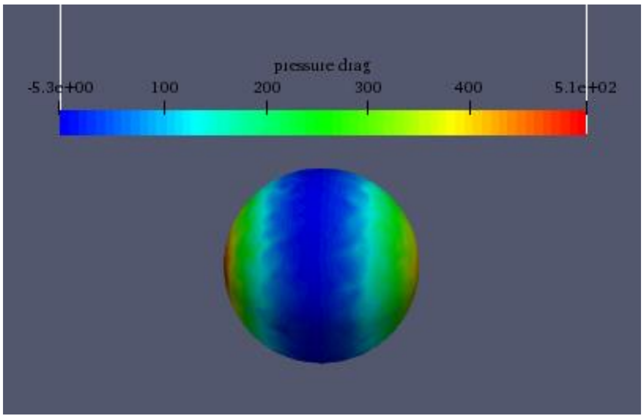

Fig e: Pressure drag of case no. 10

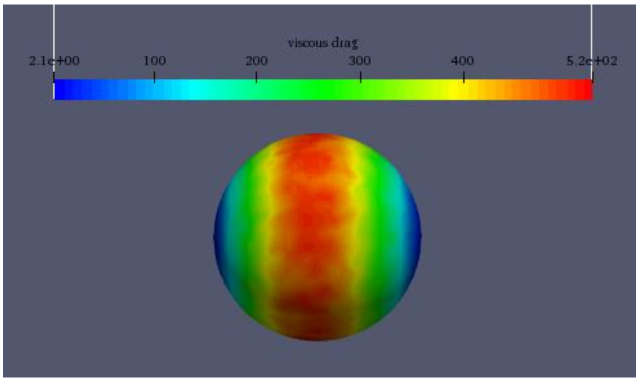

Fig f: Viscous drag of case no. 10

6. Conclusion

From the above table, it is clear that the assumptions considered for Stokes law will cause an error of about 7-10 %. This error may look negligible in the above case but can be fatal as this law is used to study the motion of microorganisms in or outside our body. It can be concluded that, for laminar flow, simpleFoam work well as seen from Fig c. Streamlines look laminar and the solution has converged well.

References

[1] *Stokes, G. G. (1851). "On the effect of internal friction of fluids on the motion of pendulums". Transactions of the Cambridge Philosophical Society. 9, part ii: 8–106.* The formula for terminal velocity (*V*) appears on p. [52], equation (127).

[2] http://openfoamwiki.net/index.php/SimpleFoam

[3] https://openfoamwiki.net/index.php/OpenFOAM_guide/The_SIMPLE_algorithm_in_OpenFOAM#The_pressure_equation